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Symmetries of sub-Riemannian surfaces

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ABSTRACT

We obtain some results on symmetries of sub-Riemannian surfaces. In case of a contact sub-Riemannian surface we base on invariants found by Hughen [15]. Using these invariants, we find conditions under which a sub-Riemannian surface does not admit symmetries. If a surface admits symmetries, we show how invariants help to find them. It is worth noting, that the obtained conditions can be explicitly checked for a given contact sub-Riemannian surface. Also, we consider sub-Riemannian surfaces which are not contact and find their invariants along the surface where the distribution fails to be contact.

0. Introduction

A sub-Riemannian manifold is a k-dimensional distribution endowed with a metric tensor on an n-dimensional manifold. At present sub-Riemannian geometry is intensively studied; this is motivated by applications in various fields of science (see, e.g. the book [1], where many applications of sub-Riemannian geometry are presented; also, for interesting examples, we refer the reader to [2–10], where applications to mechanics, thermodynamics, and biology are given). At the same time, various aspects of the theory of symmetries of sub-Riemannian manifolds are widely investigated because symmetries are always of great importance for applications [11,12]. Many papers are devoted to the theory of homogeneous (in part, symmetric) sub-Riemannian manifolds (see e.g. [13–16]). The main investigation tool in these papers is the Lie algebra theory as is usual when we study homogeneous spaces.

In the present paper we study symmetries of sub-Riemannian surfaces, i.e. of sub-Riemannian manifolds with $k = 2$ and $n = 3$. Our main goal is to give a practical tool (or an algorithmic procedure) for investigation of symmetries of a sub-Riemannian surface. The paper is organized as follows. In the first section we give in detail the construction of invariants of a contact sub-Riemannian surface using the Cartan reduction procedure (here we follow [15]) and show how to calculate them. In the second section we demonstrate how to apply invariants to finding symmetries of a contact sub-Riemannian surface. Finally, in the third section we consider a sub-Riemannian surface without assumption that it is contact and find invariants along the "singular surface", where the distribution fails to be contact.

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